Safety Transformer Benchmark

Part I - Analytical Model & Continuous Variables

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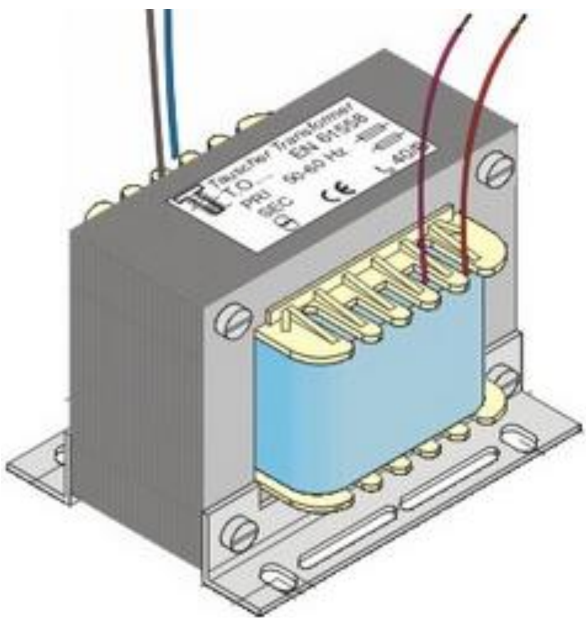
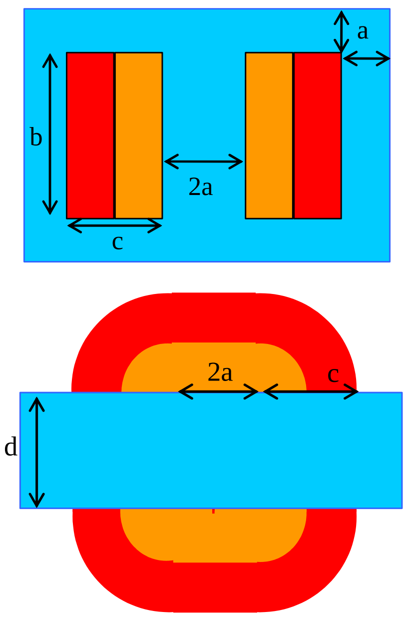




Motivations

Many analytical test functions are available in the literature to compare optimization algorithms. They exhibit some interesting features such as:

- explicit equations
- fast to compute
- known minimum
- scaled design variables and objectives
- no constraint

As algorithms' performances are changing a lot depending on the optimization problem and the model, the benchmark proposed here is intended to be representative of design (pre-sizing) problems in electrical engineering and more precisely in electromagnetic devices.

This benchmark exhibits other interesting features such as:

- multi-physics
- implicit equations
- highly constrained
- badly scaled design variables, objectives, and constraints
- multimodal, i.e. multiple minima

Links to fully detailed materials for the understanding and use of this benchmark are provided.

Analytical Model

The physical phenomena within the transformer are thermal, electric and magnetic. They are expressed in equations that are ranked using specific algorithms. The assumptions for the analytical models are uniform distribution of induction in the iron core and no voltage drop due to the magnetizing current. The magnetic field in coils is vertical.

This model leads to an implicit system of 8 multi-physical equations and other equations solved sequentially. To address the multi-physical coupling, two multidisciplinary formulations are used. The multidisciplinary feasible (MDF) formulation ensures the consistency of the model and the non-linear implicit system is solved by using the fixed-point loop. As a consequence, all physics are solved several times for each model evaluation. In the individual feasible (IDF) formulation, the model is not consistent. To ensure the consistency, two additional equality constraints are used with two additional variables that link the physics. The computing time of the model is reduced as all physics are solved one time.

The model and optimization problem have been presented at the International Conference on the Computation of Electromagnetic Fields (COMPUMAG) in June 2007 and are available in the proceedings with reference: TRAN Tuan-Vu, BRISSET Stéphane, BROCHET Pascal, "A Benchmark for Multi-Objective, Multi-Level and Combinatorial Optimizations of a Safety Isolating Transformer", COMPUMAG 2007, Aachen, Germany, 06/2007.

The equations are detailed and explained here.
The equations are given here.
The equations can be computed using the Mathcad file or the Matlab function.

In order to compute objective functions and constraints for MDF formulation:

```
x = [ 13e-3 ; 50e-3 ; 17e-3 ; 43e-3 ; 640 ; 0.32e-6 ; 2.9e-6 ];
% a(m) ; b(m) ; c(m) ; d(m) ; n1 ; S1(m²) ; S2(m²)
[f,g] = safety_transformer_function(x,false,false); % mono-objective
[f,g] = safety_transformer_function(x,true,false); % bi-objective
```

where:

$$f = \begin{bmatrix} M_{tot} \\ -\eta \end{bmatrix} \quad (bi-objective) \quad or \quad f = M_{tot} \quad (mono-objective)$$

$$g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ residue - 10^{-6} \end{bmatrix} \quad (bi-objective) \quad or \quad g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ residue - 10^{-6} \\ 0.8 - \eta \end{bmatrix} \quad (mono-objective)$$

In order to compute objective functions and constraints for IDF formulation:

```
x = [ 13e-3 ; 50e-3 ; 17e-3 ; 43e-3 ; 640 ; 0.32e-6 ; 2.9e-6 ; 100 ; 1 ];
% a(m) ; b(m) ; c(m) ; d(m) ; n1 ; S1(m²) ; S2(m²) ; Tcond_IDF(°C) ; DV2_IDF(V)
[f,g,h] = safety_transformer_function(x,false,true); % mono-objective
[f,g,h] = safety_transformer_function(x,true,true); % bi-objective
```

where:

$$f \quad unchanged$$

$$g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \end{bmatrix} \quad (bi-objective) \quad or \quad g = \begin{bmatrix} T_{cond} - 120°C \\ T_{iron} - 100°C \\ \dfrac{I_{10}}{I_1} - 0.1 \\ \dfrac{\Delta V_2}{V_{20}} - 0.1 \\ f_1 - 1 \\ f_2 - 1 \\ 0.8 - \eta \end{bmatrix} \quad (mono-objective)$$

$$h = \begin{bmatrix} \dfrac{T_{cond}}{T_{cond\_IDF}} - 1 \\ \dfrac{\Delta V_2}{\Delta V_{2\_IDF}} - 1 \end{bmatrix} \quad (bi-objective \quad and \quad mono-objective)$$

Optimization Problems

The optimization problem depends on the multidisciplinary formulation. Two additional design variables and two additional equality constraints are introduced in the optimizations problems with IDF formulation.

Some stochastic algorithms may have difficulty to deal with equality constraints. If so, prefer MDF formulation.

MDF formulation

The aim is to have a device with the smallest mass of active materials while respecting some technical constraints. The fixed-point loop stops when the residue is small enough or the fixed-point loop diverges. Therefore, a constraint on the residue is added. Only 7 continuous and bounded design variables are kept for MDF formulation:

$$3\,mm \le a \le 30\,mm \qquad 14\,mm \le b \le 95\,mm \qquad 6\,mm \le c \le 40\,mm \qquad 10\,mm \le d \le 80\,mm$$
$$200 \le n_1 \le 1200 \qquad 0.15\,mm^2 \le S_1 \le 19\,mm^2 \qquad 0.15\,mm^2 \le S_2 \le 19\,mm^2$$

Mono-objective problem

$$\begin{array}{l} \min\ f \\ \text{s.t.}\quad g \le 0 \end{array} \quad \text{that is} \quad \begin{array}{ll} \min & M_{tot} \\ \text{s.t.} & \begin{array}{llll} T_{cond} \le 120°C & T_{iron} \le 100°C & \dfrac{I_{10}}{I_1} \le 0.1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 \\ f_1 \le 1 & f_2 \le 1 & \eta \ge 0.8 & residue < 10^{-6} \end{array} \end{array}$$

Bi-objective problem

The second objective is to maximize the efficiency. The constraint on the efficiency is then removed to have a widespread Pareto front.

$$\begin{array}{l} \min\ f \\ \text{s.t.}\quad g \le 0 \end{array} \quad \text{that is} \quad \begin{array}{ll} \min\ M_{tot} & \max\ \eta \\ \text{s.t.} & \begin{array}{llll} T_{cond} \le 120°C & T_{iron} \le 100°C & \dfrac{I_{10}}{I_1} \le 0.1 & residue < 10^{-6} \\ f_1 \le 1 & f_2 \le 1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 & \end{array} \end{array}$$

IDF formulation

Two additional continuous and bounded design variables are introduced with IDF formulation:

$$\begin{array}{ccc} 3\,mm \le a \le 30\,mm & 14\,mm \le b \le 95\,mm & 6\,mm \le c \le 40\,mm \\ 10\,mm \le d \le 80\,mm & 200 \le n_1 \le 1200 & 0.15mm^2 \le S_1 \le 19mm^2 \\ 0.15mm^2 \le S_2 \le 19mm^2 & 40°C \le T_{cond\_IDF} \le 400°C & 0.1V \le \Delta V_{2\_IDF} \le 24V \end{array}$$

Mono-objective problem

$$\begin{array}{l} \min\ f \\ \text{s.t.}\quad \begin{array}{l} g \le 0 \\ h = 0 \end{array} \end{array} \quad \text{that is} \quad \begin{array}{ll} \min & M_{tot} \\ \text{s.t.} & \begin{array}{lll} T_{cond} \le 120°C & T_{iron} \le 100°C & f_1 \le 1 \\ \dfrac{I_{10}}{I_1} \le 0.1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 & f_2 \le 1 \\ \eta \ge 0.8 & T_{cond\_IDF} = T_{cond} & \Delta V_{2\_IDF} = \Delta V_2 \end{array} \end{array}$$

Bi-objective problem

The second objective is to maximize the efficiency. The constraint on the efficiency is then removed to have a widespread Pareto front.

$$\begin{array}{l} \min\ f \\ \text{s.t.}\quad \begin{array}{l} g \le 0 \\ h = 0 \end{array} \end{array} \quad \text{that is} \quad \begin{array}{ll} \min\ M_{tot} & \max\ \eta \\ \text{s.t.} & \begin{array}{lll} T_{cond} \le 120°C & T_{iron} \le 100°C & f_1 \le 1 \\ \dfrac{I_{10}}{I_1} \le 0.1 & \dfrac{\Delta V_2}{V_{20}} \le 0.1 & f_2 \le 1 \\ \Delta V_{2\_IDF} = \Delta V_2 & T_{cond\_IDF} = T_{cond} & \end{array} \end{array}$$

Optimization Results

The results are the same whatever the multidisciplinary formulation is.

Mono-objective problem

The known global optimum is:

a	12.9172mm	b	50.1221mm	c	16. 6106mm	d	43.2578mm
n_1	640.771	S_1	0.324828mm²	S_2	2.91178mm²		
ΔV_{2_IDF}	1.65798V	T_{cond_IDF}	108.818°C	IDF formulation additional design variables			
M_{tot}	2.31115kg	η	0.895537	T_{cond}	108.818°C	T_{iron}	100.000°C
f_1	1.000000	f_2	1.000000	$\frac{I_{10}}{I_1}$	0.100000	$\frac{\Delta V_2}{V_{20}}$	0.0690825
$\frac{T_{cond}}{T_{cond\_IDF}}$	1.000000	$\frac{\Delta V_2}{\Delta V_{2\_IDF}}$	1.000000	IDF formulation additional equality constraints			

All constraints are fulfilled within a tolerance of 1e-6. Four inequality constraints are active.

This solution is found by SQP in fmincon (Matlab Optimization Toolbox) modified with some techniques as multi-start, and scaling of variables, objective and constraints. The results are given for 100 starting points with uniform sampling over the design space. The convergence rate is 86% and the average number of evaluations is 187. All the results of this algorithm are in the Matlab .mat file here. The inputs and outputs of the model are in the order given in the text file.

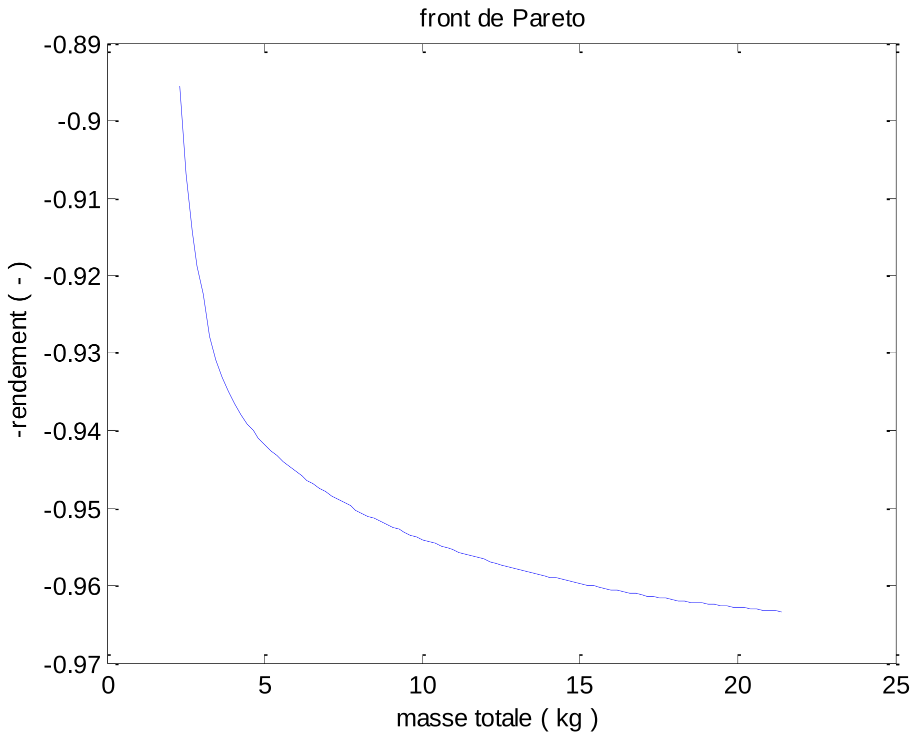
A Matlab function to run fmincon with multi-start is given here. Its gives a solution very similar and the convergence rate is less than 1%.

The command line is below and the input argument is the number of starting points (runs):

```
[xbest,fbest,convergence]=run_fmincon_on_safety_transformer_benchmark(1000)
```

Bi-objective problem

A reference Pareto front with 100 points is given:



This solution is found by SQP in Matlab fmincon modified with some techniques as multi-start, scaling of variables, objective and constraints, and epsilon-constraint transformation for multi-objective problem. The results are given for 10 starting points with uniform sampling over the design space for each of the 100 solutions of the Pareto-set, leading to 1000 mono-objective optimizations. All the results of this algorithm are in the Matlab .mat file here.

In order to draw the Pareto front, the script is below (assuming the .mat file is in Matlab current folder):

```
load('bi_objective_results.mat')
plot(bi_objective_results.graph(1,:),bi_objective_results.graph(2,:))
xlabel(bi_objective_results.axes(1))
ylabel(bi_objective_results.axes(2))
```

Contact

For any question or comment, please contact:

Dr. Stéphane BRISSET
stephane.brisset@centralelille.fr

I would be glad to see the results of your algorithms on this benchmark!